

Testing the validity of purchasing power parity in alternative markets: Evidence from the fourier quantile unit root test

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Abstract

This study analyzes the long-run validity of purchasing power parity (PPP) in three types of market economies—developed, emerging, and frontier markets—using the Fourier quantile unit root test. The analyses are conducted on 45 countries, including 10 developed, 20 developing, and 15 frontier market economies, using monthly observations from 1993:1 to 2018:8. Conventional, nonlinear, and Fourier-type unit root tests are also employed for this purpose. The Fourier quantile unit root test results provide more evidence than other tests on the validity of PPP, showing that it is valid in 8 developed, 11 emerging, and 7 frontier market economies.

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1. Introduction

Purchasing power parity (PPP) is one of the oldest and most controversial approaches in theories on the determination of exchange rates. In the broadest sense, it means that the nominal exchange rate between two currencies must be equal to the relative price in the two countries. The main idea behind PPP is that commodity prices in different countries, expressed in a common currency, should eventually equalize after arbitrage activities.

PPP is the theory of long-run equilibrium exchange rate determination. However, the inability of prices to respond to changes in nominal exchange rates invalidates this theory in

the short run. PPP theory plays an important role for economists and policy makers for the following reasons: first, it reveals whether currencies are overvalued or undervalued; second, it can be used as a general indicator of the competitiveness of tradable goods; third, it forms a substantial basis for the theories of exchange rate determination; fourth, it can determine the degree of misalignment of nominal exchange rates; fifth, it can help regulate exchange rates parity; and, sixth, it allows for a comparison of national income among different countries (Chang & Tzeng, 2011; Holmes, 2001; Sarno & Taylor, 2002).

The most common method used to test PPP is unit root analyses of real exchange rate (RER) series.¹ Thus, it

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¹ The long-run validity of PPP is analyzed in strong and weak forms (Dornbusch, 1985). The strong form of PPP is tested using unit root analyses for RER series. The weak form, however, is tested using cointegration analyses that study the existence of a long-run relationship between nominal exchange rates and relative price levels. The weak form of PPP is disregarded in this study.

investigates whether the RER series show the mean-reverting properties in the long run. If RERs contain a unit root, this means that shocks are permanent, and therefore PPP becomes invalid. In contrast, the stationarity of the RERs means that the shocks are temporary, and PPP holds in the long run.

Earlier studies that analyze the stationarity of RER series and, hence, the validity of PPP are generally based on conventional unit root tests, such as augmented Dickey-Fuller (ADF), Phillips–Perron (PP), and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS). However, the lack of power in conventional unit root tests forced researchers to use either long-span data or panel unit root techniques. At the same time, the PPP puzzle developed by Rogoff (1996) introduced the idea that the mean-reverting properties of RERs can occur with a nonlinear process. Transaction costs and government intervention in foreign exchange markets are among the reasons for RERs to have nonlinear structures (Michael et al., 1997; Taylor & Peel, 2000). This encouraged the application of nonlinear unit root tests, rather than conventional tests, in PPP analyses.²

According to Bahmani-Oskooee, Chang, Chen, and Tzeng (2017b), studies employing linear and nonlinear unit root tests focus on the mean-reverting properties of RER series, yet they ignore the effects of shocks of various magnitudes. These studies also ignore the volume and signs of shocks and simply assume that the adjustment speed of RERs to equilibrium is usually constant. However, the quantile unit root test developed by Koehler and Xiao (2004) overcomes the weaknesses mentioned above. Therefore, this study examines the long-run validity of PPP using the Fourier quantile unit root test, which was recently developed by Bahmani-Oskooee, Chang, and Ranjbar (2017a). According to the Fourier approach, it is unnecessary to assume that the break dates, the number of breaks, and break forms are known a priori. This approach is used to approximate the possible structural breaks of unknown numbers and forms, rather than directly estimating the break dates and the number of breaks (Tsong et al., 2016).

A very small number of studies have examined the validity of PPP using the Fourier quantile unit root test. Bahmani-Oskooee et al. (2017a,b) applied the Fourier quantile unit root test to 23 member countries of the Organization for Economic Cooperation and Development (OECD) and concluded that PPP is valid in 16 countries, but not Austria, Finland, France, Ireland, Portugal, and Singapore. Bahmani-Oskooee and Wu (2018) applied a quantile unit root test with smooth and sharp breaks in 34 OECD countries and found that PPP is valid in 18 countries. Mike and Kızılkaya (2019) applied the Fourier quantile unit root test in 12 emerging market economies (EMEs) and found that PPP is valid in Colombia, India, the Philippines, Poland, South Africa, and Turkey. Finally, Doğanlar et al. (2020) applied the

Fourier quantile unit root test in Turkey and its major trading partners, and the results confirmed PPP's long-run validity.

In addition to the studies mentioned, numerous empirical studies have tested the validity of PPP with different methods and country groups. Arize et al. (2010) and Bahmani-Oskooee et al. (2018) tested PPP in African countries, whereas Narayan (2005) and Cerrato and Sarantis (2008) investigated the validity of PPP in OECD countries. In addition, Chang and Tzeng (2011, 2013) and Thacker (1995) tested PPP in transition economies, and Lu et al. (2011) and Su et al. (2011) examined PPP in Latin American countries.

This study investigates the long-run validity of PPP in three types of market economies (developed, emerging, and frontier), following Morgan Stanley Capital International's country/market classification (<https://www.msci.com/market-classification/>). This paper makes three main contributions to the literature. First, it examines three types of market together for the first time—in particular, frontier market economies. Second, it allows comparisons of different unit root tests, namely, those that are conventional, nonlinear, and Fourier type. Third, it provides reliable results from the Fourier quantile unit root test, which takes into account structural breaks in the event of a nonnormal distribution. This study differs from previous studies that use the Fourier quantile unit root test, such as Bahmani-Oskooee et al. (2017a,b) which they all ignored the contributions mentioned above.

The rest of the paper is as follows. Section 2 explains the data and econometric methodology. Section 3 describes the empirical results. Section 4 presents the conclusion.

2. Data and methodology

The data used in this study are monthly observations from 1993:1 to 2018:8.^{3,4} We conducted analyses of 45 countries divided into three different types of markets: 10 developed, 20 emerging, and 15 frontier market economies. The RER series are calculated with the following equation: $q = ner + p^* - p$, where q is the real exchange rate, ner is the nominal exchange rate (i.e., the amount of national currency unit per US dollars), and p^* and p refer to the foreign consumer price index (US) and domestic consumer price index, respectively. The reason to employ the RER series, rather than real effective exchange rates, is that global trade patterns can change over time, which makes real effective exchange rate series suffer from lack of representativeness (Cenedese & Stolper, 2012). All series in the definition of the RER are expressed in logarithm. The data are collected from the International Monetary Fund's International Financial Statistics database.

The p -values of the Jarque-Bera test statistics, which are not reported, imply that all RER series, except for those for

² Studies conducted by Rogoff (1996), Sarno and Taylor (2002), Taylor (2006), and Taylor and Taylor (2004), provide important theoretical and empirical information regarding PPP.

³ This is the period for which the longest common data are available for all the countries under analysis.

⁴ See the Online Supplementary Material for further details on the econometric methodology employed in the analyses.

Table 1
Conventional/nonlinear and the Fourier-type unit root test results for developed markets.

Countries	Conventional unit root/stationarity tests			Fourier-type unit root/stationarity tests		
	ADF	KPSS	KSS	FADF	FKPSS	FKSS
Canada	-1.647 (4)	0.701 [14]*	-1.874 (4)	-2.999 (4)	0.452 [14]*	-3.578 (4)
Denmark	-2.109 (1)	0.241 [14]	-1.518 (14)	-2.635 (1)	0.658 [14]*	-2.513 (1)
Hong Kong	-2.975 (15)*	0.931 [15]*	-3.220 (15)*	-3.197 (15)	0.120 [14]	-2.592 (15)
Israel	-1.554 (13)	0.323 [14]	-1.917 (13)	-2.224 (13)	0.179 [14]*	-3.180 (13)
Japan	-1.957 (11)	1.232 [14]*	-1.936 (11)	-2.476 (11)	1.033 [14]*	-2.752 (11)
Norway	-1.544 (9)	0.216 [14]	-1.881 (9)	-1.898 (9)	0.386 [14]	-2.448 (9)
Singapore	-1.595 (15)	0.463 [15]*	-1.760 (15)	-2.199 (15)	0.244 [14]*	-1.775 (15)
Sweden	-1.344 (9)	0.501 [14]*	-1.328 (9)	-1.910 (9)	0.200 [14]	-2.583 (9)
Switzerland	-1.760 (14)	0.487 [14]*	-2.509 (10)	-3.040 (10)	0.245 [14]*	-3.235 (10)
UK	-1.565 (7)	0.485 [14]*	-1.477 (7)	-2.013 (7)	0.651 [14]*	-1.659 (7)

Note: *represents significance in 5%. The numbers in parentheses show the optimum lag length determined using recursive t-statistics. The numbers in brackets show the truncation for the Bartlett kernel.

Table 2
Fourier quantile unit root test results for developed markets.

Countries	Fourier QKS	Critical Values			k*	F-Statistics
		10%	5%	1%		
Canada	4.204*	3.009	3.267	3.818	1.2	773.27
Denmark	4.134*	3.064	3.317	3.839	1.5	254.14
Hong Kong	2.599	3.135	3.368	3.929	1.1	888.77
Israel	4.037*	3.017	3.272	3.832	1.5	444.34
Japan	2.865	3.059	3.342	3.884	0.1	201.69
Norway	3.307*	3.033	3.251	3.767	1.5	289.86
Singapore	3.850*	3.081	3.373	3.874	1.3	1132.62
Sweden	3.446*	3.105	3.362	3.832	1.7	223.66
Switzerland	4.563*	3.078	3.310	3.724	1.3	271.72
UK	3.587*	3.017	3.355	3.800	0.1	93.30

Note: k* represents optimal frequency. We used the F statistics in order to test the absence of non-linear component. We calculated the critical values of the Fourier QKS statistics by means of re-sampling procedure and 1000 replications. * represents significance in 5%.

South Africa and Tunisia, exhibit a nonnormal distribution.⁵ This is a strong justification for using the quantile approach. [Koenker and Xiao \(2004\)](#) stated that the quantile autoregressive-based unit root test is more powerful than the conventional least squares-based unit root tests in cases of departure from the Gaussian residuals.

3. Empirical results

This study investigates the mean-reverting properties of RER series using three different unit root/stationarity tests, respectively: (1) conventional and nonlinear unit root tests, including ADF, KPSS, and KSS tests; (2) Fourier-type unit root tests that include the Fourier-KPSS (FKPSS; [Becker et al., 2006](#)), Fourier-ADF (FADF; [Christopoulos & León-Ledesma, 2010](#)), and Fourier-KSS (FKSS; [Christopoulos & León-Ledesma, 2010](#)) tests; and (3) the Fourier quantile unit root test developed by [Bahmani-Oskooee et al. \(2017a,b\)](#). We focus mainly on the Fourier quantile unit root test results. However, the conventional/nonlinear and the Fourier-type

unit-root tests are included to provide a comparison. Next, the results of these tests are reported individually for each type of market.

3.1. Developed markets

[Table 1](#) presents the conventional/nonlinear and the Fourier-type unit root test results for developed market economies. The ADF and the KSS unit root tests reject the null of a unit root only in Hong Kong. According to the KPSS test, the null of stationarity is not rejected in Denmark, Israel, and Norway. However, the FKPSS stationarity test shows that RER series are stationary for three countries: Hong Kong, Norway, and Sweden. The FADF and the FKSS unit root test results imply that the RER series are not stationary for all countries. The results from conventional/nonlinear and the Fourier-type unit root tests reveal that the RER series do not show mean-reverting properties in most of the developed market economies.

[Table 2](#) reports the Fourier quantile unit root test results for developed market economies. The QKS test offers a general perspective about the mean-reverting properties of each RER series. Accordingly, the null of a unit root is rejected for Canada, Denmark, Israel, Norway, Singapore, Sweden,

⁵ To conserve space, summary statistics of the RER series are not presented in the text, but they are available from the authors upon request.

Switzerland, and the United Kingdom, and PPP is valid for these countries. The Fourier quantile unit root test results provide more evidence than the others for the stationarity of the RER series.

Table 3 shows the RER behaviors in each specific quantile for developed market economies. $\alpha_0(\tau)$ and $\rho_1(\tau)$ for eight developed economies, which have QKS statistics that are significant at the 5 percent level, are important parameters in our evaluation. $\alpha_0(\tau)$ refers to the size of the shocks to each quantile. Accordingly, the RER in Norway has the smallest shock (−0.031), whereas the RER in Sweden has the largest (0.028). The RER in Singapore shows the smallest variation in shocks, from −0.017 to 0.017. In addition, $\rho_1(\tau)$ plays a key role in determining the mean reversion of RER series in each quantile. According to the *p*-values of $\rho_1(\tau)$, we found different RER behavior in eight developed countries. First, the RER series in Denmark, Norway, and Singapore show stationary behavior in all quantiles. Second, the RER series in Canada, Sweden, and the UK show unit root behavior in high quantiles. Third, the RER series in Israel and Switzerland show unit root behavior in low quantiles.

When we evaluate $\alpha_0(\tau)$ and $\rho_1(\tau)$ together for these countries, we see that negative shocks to RER series have transitory effects, and positive shocks have permanent effects

in Canada, Sweden, and the UK in the long run. In contrast, in Israel and Switzerland, positive shocks to the RER series have only temporary effects, whereas negative shocks have permanent effects. The unit root behaviors in these countries are presented in Figure S1 (see Online Supplementary Material).

3.2. Emerging market economies

In this section, we apply a process to developed market economies similar to that for EMEs. Table 4 presents the conventional/nonlinear and the Fourier-type unit root test results for EMEs. The ADF test does not reject the null of a unit root for all countries. According to the KPSS stationarity test, the null of stationarity is not rejected for Brazil, Chile, Colombia, Egypt, Indonesia, Korea, Mexico, the Philippines, and Thailand. The KSS test rejects the null of a unit root for Indonesia, Korea, Mexico, Thailand, and Turkey. However, the FKPSS test results show that RER series are stationary for Hungary, Korea, Malaysia, and Poland. According to the FADF test, the null of a unit root is rejected for China, India, and Poland. Finally, the FKSS test results show that the null of a unit root is rejected for Brazil, Egypt, Indonesia, Korea, Malaysia, Mexico, South Africa, and Thailand. The results of the conventional/nonlinear and the Fourier-type unit root tests

Table 3
Intercepts $\alpha_0(\tau)$ and autoregressive coefficients $\rho_1(\tau)$ for developed markets.

Countries	τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Canada	$\alpha_0(\tau)$	−0.020	−0.014	−0.009	−0.004	0.001	0.004	0.008	0.012	0.017
	<i>p</i> -value	0.000*	0.000*	0.000*	0.002*	0.303	0.000*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.852	0.887	0.899	0.894	0.928	0.951	0.963	0.951	0.947
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.002*	0.015*	0.079	0.021*	0.084
Denmark	$\alpha_0(\tau)$	−0.026	−0.018	−0.013	−0.006	0.000	0.005	0.011	0.016	0.027
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.429	0.004*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.911	0.897	0.902	0.919	0.923	0.944	0.946	0.942	0.922
	<i>p</i> -value	0.004*	0.000*	0.000*	0.001*	0.003*	0.023*	0.019*	0.033*	0.025*
Israel	$\alpha_0(\tau)$	−0.019	−0.012	−0.008	−0.005	−0.001	0.003	0.007	0.012	0.018
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.139	0.008*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.954	0.919	0.903	0.920	0.876	0.887	0.879	0.881	0.842
	<i>p</i> -value	0.136	0.002*	0.000*	0.002*	0.000*	0.000*	0.000*	0.001*	0.000*
Norway	$\alpha_0(\tau)$	−0.031	−0.021	−0.009	−0.004	0.000	0.005	0.010	0.018	0.026
	<i>p</i> -value	0.000*	0.000*	0.000*	0.004*	0.402	0.001*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.918	0.910	0.921	0.929	0.923	0.940	0.930	0.918	0.904
	<i>p</i> -value	0.023*	0.009*	0.007*	0.003*	0.001*	0.014*	0.005*	0.004*	0.006*
Singapore	$\alpha_0(\tau)$	−0.017	−0.010	−0.007	−0.004	−0.001	0.001	0.006	0.010	0.017
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.220	0.068	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.887	0.933	0.931	0.917	0.882	0.872	0.906	0.916	0.885
	<i>p</i> -value	0.007*	0.028*	0.015*	0.003*	0.000*	0.000*	0.002*	0.023*	0.018*
Sweden	$\alpha_0(\tau)$	−0.029	−0.018	−0.011	−0.004	0.000	0.006	0.012	0.018	0.028
	<i>p</i> -value	0.000*	0.000*	0.000*	0.018*	0.414	0.001*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.939	0.914	0.917	0.941	0.949	0.957	0.955	0.958	0.955
	<i>p</i> -value	0.058	0.005*	0.001*	0.011*	0.024*	0.040*	0.033*	0.062	0.103
Switzerland	$\alpha_0(\tau)$	−0.029	−0.019	−0.011	−0.004	0.000	0.006	0.012	0.017	0.025
	<i>p</i> -value	0.000*	0.000*	0.000*	0.009*	0.401	0.000*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.930	0.923	0.907	0.902	0.902	0.887	0.919	0.941	0.929
	<i>p</i> -value	0.055	0.010*	0.002*	0.001*	0.000*	0.000*	0.001*	0.018*	0.018
UK	$\alpha_0(\tau)$	−0.024	−0.015	−0.009	−0.005	−0.001	0.005	0.008	0.014	0.025
	<i>p</i> -value	0.000*	0.000*	0.000	0.000*	0.297	0.001*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.908	0.934	0.959	0.953	0.956	0.981	0.987	0.970	0.954
	<i>p</i> -value	0.005*	0.007*	0.032*	0.012*	0.029	0.229	0.301	0.162	0.100

Note; *represents significance in 5%.

Table 4
Conventional/nonlinear and Fourier-type unit root/stationarity test results for EMEs.

Countries	Conventional unit root/stationarity tests			Fourier-type unit root/stationarity tests		
	ADF	KPSS	KSS	FADF	FKPSS	FKSS
Brazil	-1.702 (14)	0.249 [14]	-2.310 (14)	-1.965 (10)	0.913 [14]*	-4.493 (5)*
Chile	-1.893 (11)	0.204 [14]	-1.780 (11)	-1.938 (13)	0.533 [14]*	-1.979 (13)
China	-1.457 (0)	1.468 [14]*	-1.730 (0)	-7.691 (12)*	0.377 [14]*	-2.654 (0)
Colombia	-2.036 (2)	0.452 [14]	-1.728 (2)	-1.910 (7)	1.297 [14]*	-1.665 (7)
Czech Rep.	-1.723 (7)	1.602 [14]*	-2.528 (7)	-3.423 (7)	0.225 [14]*	-2.302 (7)
Egypt	-1.800 (4)	0.275 [14]	-1.648 (4)	-2.366 (3)	0.627 [14]*	-5.174 (4)*
Hungary	-1.475 (7)	1.169 [14]*	-2.263 (7)	-3.262 (7)	0.085 [14]	-3.101 (7)
India	-0.870 (13)	1.754 [14]*	-1.873 (4)	-4.025 (9)*	0.418 [14]*	-3.200 (9)
Indonesia	-2.526 (15)	0.281 [14]	-7.692 (15)*	-3.041 (15)	0.366 [14]*	-7.826 (15)*
Korea	-2.779 (8)	0.194 [14]	-3.895 (8)*	-3.149 (8)	0.186 [14]	-4.465 (8)*
Malaysia	-2.071 (9)	0.603 [14]*	-2.096 (8)	-2.564 (9)	0.396 [14]	-7.956 (9)*
Mexico	-2.316 (8)	0.366 [14]	-4.101 (8)	-3.594 (8)	0.203 [14]*	-4.683 (8)*
Pakistan	-1.591 (11)	0.497 [14]*	-1.648 (11)	-2.416 (11)	0.244 [14]*	-2.037 (11)
Peru	-1.359 (3)	0.541 [14]*	-1.523 (3)	-3.036 (9)	0.173 [14]*	-3.446 (9)
Philippines	-1.624 (8)	0.395 [14]	-1.495 (8)	-2.472 (8)	0.319 [14]*	-2.146 (8)
Poland	-2.179 (9)	1.066 [14]*	-2.024 (9)	-4.303 (6)*	0.066 [14]	-3.062 (7)
S. Africa	-2.109 (8)	0.755 [14]*	-2.441 (8)	-2.702 (8)	0.482 [14]*	-3.531 (8)*
S. Arabia	-1.750 (13)	0.466 [15]*	-1.955 (13)	-1.995 (13)	0.262 [14]*	-3.062 (13)
Thailand	-1.734 (10)	0.356 [14]	-5.610 (7)*	-2.155 (10)	0.466 [14]*	-8.701 (15)*
Turkey	-0.851 (12)	1.003 [14]*	-3.365 (15)*	-2.661 (12)	0.192 [13]*	-3.161 (12)

Note: *represents significance in 5%. The numbers in parentheses show the optimum lag length determined using recursive t-statistics. The numbers in brackets show the truncation for the Bartlett kernel.

Table 5
Fourier quantile unit root test results for EMEs.

Countries	Fourier QKS	Critical Values			K*	F-Statistics
		10%	5%	1%		
Brazil	4.878*	2.810	3.138	3.779	1.6	356.82
Chile	4.167*	2.990	3.248	3.683	1.7	320.02
China	2.669	3.072	3.406	4.147	0.8	469.21
Colombia	2.854	3.059	3.337	3.790	0.9	874.67
Czech Rep.	2.296	2.964	3.270	3.735	1.7	195.49
Egypt	3.768*	2.593	2.946	3.415	1.7	473.36
Hungary	2.496	2.929	3.214	3.884	1.1	728.49
India	4.018*	2.976	3.241	3.612	0.8	907.96
Indonesia	2.669	2.424	2.679	3.155	1.3	171.97
Korea	4.355*	2.896	3.154	3.599	2.5	80.53
Malaysia	3.171*	2.708	3.005	3.667	1.6	279.14
Mexico	5.064*	2.821	3.116	3.578	0.1	77.752
Pakistan	2.089	2.89	3.176	3.768	1.2	695.49
Peru	2.659	2.944	3.170	3.597	1.3	501.69
Philippines	3.581*	2.95	3.160	3.598	1.4	975.33
Poland	3.259	3.043	3.291	3.84	1	369.01
S. Africa	4.652*	3.012	3.265	3.955	1.8	196.46
S. Arabia	3.564*	2.982	3.240	3.872	1.3	2979.78
Thailand	2.654	2.747	3.005	3.661	1.4	508.80
Turkey	4.901*	2.871	3.166	3.742	1.1	579.82

Note: k* represents optimal frequency. We used the F statistics in order to test the absence of non-linear component. We calculated the critical values of the Fourier QKS statistics by means of re-sampling procedure and 1000 replications. * represents significance in 5%.

(except for ADF and FADF) reveal that RER series show mean-reverting properties only for Indonesia, Korea, Mexico, and Thailand.

Table 5 reports the Fourier quantile unit root test results for EMEs. According to the QKS test results, the null of a unit root is rejected for 11 out of 20 EMEs: Brazil, Chile, Egypt,

Table 6
Intercepts $\alpha_0(\tau)$ and autoregressive coefficients $\rho_1(\tau)$ for EMEs.

Countries	τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Brazil	$\alpha_0(\tau)$	-0.039	-0.026	-0.015	-0.008	-0.004	0.002	0.008	0.020	0.042
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.015*	0.230	0.001*	0.000*	0.000*
	$\rho_1(\tau)$	0.884	0.900	0.929	0.954	0.969	0.988	0.993	1.006	1.031
Chile	$\alpha_0(\tau)$	-0.026	-0.018	-0.010	-0.004	-0.001	0.003	0.007	0.016	0.025
	<i>p</i> -value	0.000*	0.000*	0.000*	0.001*	0.190	0.003*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.905	0.929	0.950	0.947	0.952	0.963	0.956	0.928	0.939
Egypt	$\alpha_0(\tau)$	-0.017	-0.011	-0.007	-0.005	-0.003	0.000	0.004	0.007	0.012
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.002*	0.414	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	1.017	1.019	1.015	1.007	0.994	0.985	0.963	0.954	0.929
India	$\alpha_0(\tau)$	-0.019	-0.013	-0.009	-0.003	0.000	0.004	0.006	0.011	0.018
	<i>p</i> -value	0.000*	0.000*	0.000*	0.006*	0.445	0.000*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.982	0.950	0.951	0.932	0.933	0.920	0.903	0.888	0.842
Korea	$\alpha_0(\tau)$	-0.023	-0.014	-0.011	-0.006	-0.002	0.001	0.007	0.012	0.023
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.107	0.158	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.934	0.941	0.943	0.946	0.960	0.969	0.973	0.968	0.972
Malaysia	$\alpha_0(\tau)$	-0.015	-0.009	-0.005	-0.003	-0.001	0.001	0.004	0.008	0.019
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.085	0.089	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.951	0.953	0.971	0.980	0.988	0.998	1.011	1.042	1.057
Mexico	$\alpha_0(\tau)$	-0.025	-0.018	-0.012	-0.008	-0.004	0.002	0.008	0.016	0.034
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.015*	0.145	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.891	0.912	0.934	0.937	0.940	0.939	0.961	0.971	0.936
Philippines	$\alpha_0(\tau)$	-0.017	-0.011	-0.008	-0.005	-0.001	0.002	0.007	0.012	0.017
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.112	0.070	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.919	0.962	0.968	0.980	0.987	0.986	0.923	0.887	0.890
S. Africa	$\alpha_0(\tau)$	-0.035	-0.024	-0.015	-0.008	-0.002	0.003	0.014	0.022	0.041
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.161	0.127	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.893	0.912	0.926	0.950	0.972	0.987	0.993	0.974	0.969
S. Arabia	$\alpha_0(\tau)$	-0.005	-0.003	-0.002	-0.001	0.001	0.002	0.002	0.004	0.005
	<i>p</i> -value	0.000*	0.000*	0.000*	0.050	0.013*	0.000*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.910	0.919	0.921	0.929	0.934	0.929	0.927	0.941	0.958
Turkey	$\alpha_0(\tau)$	-0.034	-0.022	-0.015	-0.010	-0.003	0.002	0.008	0.023	0.040
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.031*	0.153	0.001*	0.000*	0.000*
	$\rho_1(\tau)$	0.818	0.839	0.895	0.919	0.937	0.932	0.956	0.978	0.958
	<i>p</i> -value	0.000*	0.000*	0.000*	0.001*	0.007*	0.006*	0.111	0.309	0.233

Note: *represents significance in 5%.

India, Korea, Malaysia, Mexico, the Philippines, South Africa, Saudi Arabia, and Turkey. Thus, PPP is valid for these countries. These results show that the Fourier quantile unit root test provides more evidence than the other tests for the stationarity of RER series, as in developed market economies.

Table 6 shows the RER behaviors in each specific quantile for 11 EMEs that have significant QKS statistics at the 5 percent level. Based on the value of $\alpha_0(\tau)$, the RER of Brazil has the smallest shocks (-0.039) and the largest (0.042), whereas the RER of Saudi Arabia has the smallest variation in shocks, from -0.005 to 0.005. However, the behavior of RER series differs across the countries. According to the *p*-values of $\rho_1(\tau)$, the RER series of Brazil, Korea, Malaysia, Mexico, South Africa, and Turkey show stationary behavior in the low

quantiles. Second, the RER series of Egypt and India show stationary behavior in the high quantiles. Third, the RER series of the Philippines show unit root behavior in the middle quantiles. Finally, the RER series of Chile and Saudi Arabia show stationary behavior in almost all the quantiles.

When we evaluate $\alpha_0(\tau)$ and $\rho_1(\tau)$ together for these countries, we see that negative shocks to the RER series have transitory effects, whereas positive shocks have permanent effects in the long run for Brazil, Korea, Malaysia, Mexico, South Africa, Saudi Arabia, and Turkey. Conversely, for Egypt, India, and the Philippines, positive shocks to the RER series have only transitory effects, whereas negative shocks have permanent effects. The unit root behavior for these countries is presented in Figure S2 (see Online Supplementary Material).

Table 7
Conventional/nonlinear and Fourier type unit root/stationarity test results for frontier markets.

Countries	Conventional unit root/stationarity tests			Fourier-type unit root/stationarity tests		
	ADF	KPSS	KSS	FADF	FKPSS	FKSS
Botswana	-2.313 (1)	0.174 [14]	-2.058 (1)	-2.885 (1)	0.278 [14]	-1.616 (1)
Bulgaria	-1.596 (10)	1.782 [14]*	-4.126 (10)*	-2.809 (10)	0.523 [14]*	-3.333 (10)
Croatia	-2.481 (1)	0.698 [14]*	-2.868 (1)	-3.233 (1)	0.114 [14]	-3.083 (1)
Ghana	-1.447 (14)	0.833 [14]*	-3.835 (12)*	-2.361 (14)	0.736 [14]*	-2.668 (14)
Jamaica	-2.294 (2)	1.162 [14]*	-1.765 (2)	-2.379 (13)	1.495 [14]*	-3.214 (13)
Jordan	0.124 (10)	1.775 [14]*	-0.321 (3)	-1.640 (3)	0.641 [14]*	-1.317 (3)
Kenya	-0.796 (1)	1.984 [14]*	-1.697 (1)	-1.954 (1)	0.626 [14]*	-1.555 (1)
Mauritius	-1.853 (7)	0.548 [14]*	-1.890 (7)	-3.503 (6)	0.190 [14]*	-4.207 (6)*
Morocco	-2.094 (12)	0.195 [14]	-2.268 (12)	-3.036 (12)	0.281 [14]	-2.761 (1)
Nigeria	-2.095 (0)	0.185 [14]	-3.297 (0)*	-2.053 (0)	0.237 [14]	-4.943 (0)*
Romania	-1.699 (13)	1.504 [14]*	-4.222 (8)*	-3.152 (8)	0.217 [13]*	-4.471 (14)*
Sri Lanka	-0.931 (12)	1.431 [15]*	-1.679 (12)	-2.872 (12)	0.214 [14]*	-3.721 (11)*
Trinidad and Tobago	-0.111 (12)	1.895 [15]*	-0.581 (12)	-1.519 (12)	0.784 [14]*	-1.194 (12)
Tunisia	0.035 (12)	1.492 [14]*	0.492 (12)	-0.216 (12)	1.396 [14]*	1.173 (12)
Ukraine	-1.348 (14)	0.220 [13]	-4.497 (14)*	-1.851 (13)	0.508 [13]*	-5.268 (13)*

Note: * represents significance in 5%. The numbers in parentheses show the optimum lag length determined using recursive t-statistics. The numbers in brackets show the truncation for the Bartlett kernel.

Table 8
Fourier quantile unit root test results for frontier markets.

Countries	Fourier QKS	Critical Values			K*	F-Statistics
		10%	5%	1%		
Botswana	3.665*	2.993	3.194	3.671	1.4	164.12
Bulgaria	6.056*	3.023	3.262	3.881	0.8	1382.82
Croatia	2.545	3.139	3.457	3.922	1.1	165.00
Ghana	1.719	2.877	3.158	3.636	1.9	298.14
Jamaica	2.529	2.846	3.088	3.673	0.1	219.31
Jordan	1.544	2.989	3.272	3.821	0.7	1360.92
Kenya	2.928*	2.671	2.879	3.355	0.7	1435.67
Mauritius	4.148*	2.843	3.133	3.787	1.2	341.40
Morocco	3.721*	2.996	3.275	3.757	1.8	274.62
Nigeria	1.175	2.481	2.743	3.365	1.6	108.77
Romania	2.391	2.939	3.209	3.783	0.9	780.31
Sri Lanka	2.470	2.905	3.141	3.656	0.9	764.07
Trinidad and Tobago	4.14*	2.918	3.282	3.877	0.6	6674.96
Tunisia	1.713	3.086	3.334	3.791	0.1	352.40
Ukraine	3.409*	2.551	2.830	3.614	2.1	61.28

Note: k* represents optimal frequency. We used the F statistics in order to test the absence of non-linear component. We calculated the critical values of the Fourier QKS statistics by means of re-sampling procedure and 1000 replications. * represents significance in 5%.

3.3. Frontier market economies

In this section, we apply to frontier market economies the same process as discussed above for developed and emerging market economies. Table 7 illustrates the conventional/nonlinear and the Fourier-type unit root test results for frontier markets. The ADF and the FADF unit root test results show that the RER series are not stationary for all countries. According to the KPSS stationarity tests, the null of stationarity is not rejected for Botswana, Morocco, Nigeria, and Ukraine. The KSS unit root test rejects the null of a unit root for Bulgaria, Ghana, Nigeria, Romania, and Ukraine. However,

the FKPSS stationarity test does not reject the null of stationarity for Botswana, Croatia, Morocco, and Nigeria. The FKSS unit root test shows that RER series are stationary for Mauritius, Nigeria, Romania, Sri Lanka, and Ukraine. When the results of conventional/nonlinear and the Fourier-type unit root tests are evaluated together, we see that the RER series of only Nigeria and Ukraine show mean-reverting properties.

Table 8 presents the Fourier quantile unit root test results for frontier market economies. Fourier QKS test results show that the null of a unit root is rejected for Botswana, Bulgaria, Kenya, Mauritius, Morocco, Trinidad and Tobago, and Ukraine. Therefore, PPP is valid for these countries. The

Table 9
Intercepts $\alpha_0(\tau)$ and autoregressive coefficients $\rho_1(\tau)$ for frontier markets.

Countries	τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Botswana	$\alpha_0(\tau)$	-0.028	-0.016	-0.012	-0.007	-0.002	0.004	0.010	0.017	0.026
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.195	0.031*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.942	0.948	0.942	0.945	0.972	0.966	0.945	0.951	0.945
Bulgaria	$\alpha_0(\tau)$	-0.033	-0.022	-0.015	-0.007	0.000	0.005	0.012	0.018	0.029
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.473	0.003*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.934	0.936	0.944	0.908	0.911	0.892	0.845	0.864	0.863
Kenya	$\alpha_0(\tau)$	-0.026	-0.015	-0.007	-0.003	0.000	0.004	0.007	0.015	0.029
	<i>p</i> -value	0.000*	0.000*	0.000*	0.009*	0.450	0.001*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.995	0.982	1.010	1.002	0.992	0.981	0.958	0.935	0.887
Mauritius	$\alpha_0(\tau)$	-0.019	-0.010	-0.007	-0.004	-0.001	0.003	0.005	0.009	0.019
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.138	0.001*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.957	0.954	0.939	0.935	0.925	0.932	0.925	0.918	0.846
Morocco	$\alpha_0(\tau)$	-0.020	-0.014	-0.008	-0.004	-0.001	0.003	0.008	0.013	0.021
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.292	0.007*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.887	0.905	0.900	0.903	0.905	0.923	0.905	0.883	0.853
Trinidad and Tobago	$\alpha_0(\tau)$	-0.011	-0.006	-0.003	-0.001	0.001	0.002	0.004	0.007	0.010
	<i>p</i> -value	0.000*	0.000*	0.000*	0.067	0.170	0.000*	0.000*	0.000*	0.000*
	$\rho_1(\tau)$	0.990	1.004	0.974	0.973	0.967	0.962	0.948	0.917	0.906
Ukraine	$\alpha_0(\tau)$	-0.029	-0.020	-0.014	-0.010	-0.006	-0.001	0.006	0.014	0.032
	<i>p</i> -value	0.000*	0.000*	0.000*	0.000*	0.001*	0.329	0.024*	0.001*	0.003*
	$\rho_1(\tau)$	0.951	0.962	0.975	0.980	0.985	0.987	1.004	1.018	1.022
	<i>p</i> -value	0.024*	0.001*	0.005*	0.023*	0.120	0.207	0.421	0.251	0.361

Note: *represents significance in 5%.

results show that, just as in developed and emerging market economies, the Fourier quantile unit root test provides more evidence than the other tests for the stationarity of RER series in frontier markets.

Table 9 shows the RER behaviors in each specific quantile for seven frontier market economies that have significant QKS statistics at the 5 percent level. Based on $\alpha_0(\tau)$, the RER of Bulgaria has the smallest shocks (-0.033) and the RER of Ukraine has the largest (0.032). The RER of Trinidad and Tobago had the smallest variations in shocks, from -0.011 to 0.010. However, according to the *p*-values of $\rho_1(\tau)$, the RER series of Ukraine show stationary behavior in the low quantiles. Second, the RER series of Bulgaria, Kenya, Mauritius, and Trinidad and Tobago show stationary behavior in the high quantiles. Third, the RER series of Morocco show stationary behavior in all quantiles. Fourth, the RER series of Botswana show unit root behavior in the middle quantiles.

When we evaluate $\alpha_0(\tau)$ and $\rho_1(\tau)$ together for these countries, we see that negative shocks to the RER series for Ukraine have transitory effects, whereas positive shocks have permanent effects in the long run. In contrast, positive shocks to the RER series have only transitory effects, whereas negative shocks have permanent effects for Bulgaria, Kenya, Mauritius, and Trinidad and Tobago. The unit-root behaviors for these countries are presented in Figure S3 (see Online Supplementary Material).

4. Conclusion

This study examines the long-run validity of PPP by applying the Fourier quantile unit root test to three types of market economies. Additionally, we conduct conventional/nonlinear and Fourier-type unit root tests to compare the results of various tests. Conventional/nonlinear and the Fourier-type unit root test results do not support the validity of PPP for most of the countries studied. However, the results of the Fourier quantile unit root test confirm the validity of PPP in the long run for 26 out of the 45 countries under analysis. In conclusion, the Fourier quantile unit root test finds more evidence in favor of the PPP hypothesis than the conventional/nonlinear and the Fourier-type unit root tests.

This study has some crucial policy implications. Frontier market economies mostly have negative shocks, where currency appreciation can exacerbate their current account imbalances. Therefore, these countries should avoid expansionary macroeconomic policies that could lead to inflation. Conversely, EMEs mostly experience positive shocks, where currency depreciation can help them reduce current account imbalances. Therefore, monetary and fiscal policies pursued by these countries should not be inflationary, otherwise depreciation will be eroded. Another policy implication is that countries where PPP is valid can employ this approach to determine whether their currencies are

misaligned. Finally, the Fourier quantile unit root test is more reliable when the series contain structural breaks and do not have a normal distribution. Therefore, researchers can obtain more evidence on the long-run validity of PPP when they use the Fourier quantile unit root test. This work can be extended by defining the RER series in terms of the euro and other major currencies to test the long-run validity of PPP for these countries.

Declaration of competing interest

The authors declare that there is no conflict of interest for this research.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.bir.2020.12.004>.

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